Five Year Integrated M.Sc. Examination, 2024
Semester - V
Code: PH-3-5-5

Time: Three Hours | Subject: Statistical Mechanics | Full Marks: 40

Questions are of values as indicated in the margin
Answer question number **one** and any **three** from the rest

1. Answer **any five** questions

$5 \times 2 = 10$

(a) "Quantum identical particles are indistinguishable"- Justify .

(b) Define microcanonical ensemble. What are the differences between microcanonical and grand canonical ensembles?

(c) Show that the canonical partition function for a non-interacting N particles system can be written as a N-th power of single particle partition function.

(d) How does the number of accessible state Ω change in case of a reversible and an irreversible process?

(e) Consider a system with 10 degenerate energy levels. How many way one can distribute eight Bosonic particles among these levels?

(f) What is the dimension of the phase space of a diatomic molecule confined in two dimensions? Explain your answer.

(g) State and explain the postulate of equal a priori probability.

2. (a) A particle performs random walk motion in one dimension about the origin O, and at each step it moves by a unit distance either to the left or to the right. Suppose the probability of its being to the right is p, while the probability of its being to the left is $q = 1 - p$. Show that after a total N steps the particle will be found at n steps right to the origin (O) with the probability distribution

$$W_N(n) = \frac{N!}{n!(N-n)!}\, p^n (1-p)^{N-n}\,.$$

(b) Displacement is defined as $m = 2n_1 - N$. Calculate mean displacement $(\bar{m})$ of the particle from the origin after N number of steps.

5+5=10

3. (a) Consider Brownian motion of particles with mass m moving under the influence of Brownian force $\vec{F}_B$. Particles are also subject to viscous drag force with drag coefficient μ. Write the equation of motion for the particle.

   (b) Show that, for Brownian motion,

$$\frac{d}{dt}\vec{x}.\vec{v} = \vec{v}.\vec{v} - \frac{\mu}{m}\vec{x}.\vec{v} + \frac{1}{m}\vec{x}.\vec{F}_B \,,$$

   where, $\vec{x}$ and $\vec{v}$ are the position and velocity of the particle, respectively. Solve this equation for average value $\langle\vec{x}.\vec{v}\rangle$ to prove

$$\langle\vec{x}.\vec{v}\rangle = \frac{m}{\mu}\langle\vec{v}^{\,2}\rangle\left(1 - e^{-\frac{\mu t}{m}}\right)\,,$$

   where, $\langle\vec{v}^{\,2}\rangle$ is the average of $\vec{v}^{\,2}$.

   (c) Show that the entropy in a canonical ensemble is

$$S = k_B(\ln Z + \beta\bar{E})\,,$$

   where Z is canonical partition function and $\bar{E}$ is the average energy.

2+(3+3)+2=10

4. (a) An non-interacting system has two energy states E_1 and E_2 ($E_2 > E_1$), with populations n_1 and n_2, respectively ($n_1, n_2 >> 1$). The systems is in contact with a heat bath at temperature T. Calculate the average energy and entropy of the system.

   (b) Show that the canonical entropy matches with microcanonical entropy at thermodynamic limit.

   (c) Consider a system of two particles, each of which can be in any of three respective energies 0, ϵ and 3ϵ. The system is in contact with a heat reservoir at temperature T. Calculate the canonical partition function (i) if the two particles are Fermions of same type, (ii) if the two particles are Bosons of same type.

(2+1)+3+(2+2)=10

5. (a) Briefly state the initial assumptions required to derive canonical partition function.

   (b) Derive the partition function for canonical ensemble.

   (c) The number of states (Ω) accessible to a system of N molecules of ideal gas of energy E, confined in volume V is of the form $\Omega(N, V, E) \propto V^N f(E)$. From this relation, find the equation of state of ideal gas.

1+5+4=10